

Part 1: Dimension-less Modelisation

The 2D system is

$$\begin{cases} \frac{dX}{dt} = \frac{aX}{a_0 + X} - \frac{bXY}{b_0 + X} \\ \frac{dY}{dt} = \frac{cXY}{c_0 + XY} - dY \end{cases}$$

We set $U = X/a_0$ which yields

$$\begin{cases} a_0 \frac{dU}{dt} = \frac{aU}{1+U} - \frac{bUY}{B_0 + U} \\ \frac{dY}{dt} = \frac{cUY}{c_0/a_0 + UY} - dY \end{cases} \quad \text{with } B_0 = b_0/a_0$$

Now we set $V = Y/\gamma_0$ with $\gamma_0 = c_0/a_0$ which yields

$$\begin{cases} a_0 \frac{dU}{dt} = \frac{aU}{1+U} - \frac{b\gamma_0 UV}{B_0 + U} \\ \gamma_0 \frac{dV}{dt} = \frac{cUV}{1+UV} - d\gamma_0 V \end{cases}$$

Finally we change the time scale $s = at/a_0$ and obtain the dimension-less system

$$\begin{cases} (1) \Leftrightarrow \frac{dU}{ds} = \frac{U}{1+U} - \frac{BUV}{B_0 + U} \\ (2) \Leftrightarrow \frac{dV}{ds} = \frac{CUV}{1+UV} - DV \end{cases}$$

where the new parameters can be expressed as function of the old ones:

$$\begin{cases} B = b\gamma_0/a = bc_0/(aa_0) \\ B_0 = b_0/a_0 \\ C = ca_0/(a\gamma_0) = ca_0^2/(ac_0) \\ D = a_0d/a \end{cases}$$

The new version of the model has only 4 parameters – against the previous 7, which is going to make the theoretical study far easier. We use the dimension-less version of the Dynamic System in the rest of the study.